Optical and Quantum Electronics (2023) 55:991
https://doi.org/10.1007/s11082-023-05236-3

Investigation of travelling wave solutions for the (3 + 1)-dimensional hyperbolic nonlinear Schrödinger equation using Riccati equation and F-expansion techniques

Mohamed R. Ali[1,2] · **Mahmoud A. Khattab**[1] · **S. M. Mabrouk**[3]

Received: 14 December 2022 / Accepted: 27 July 2023

Abstract

The (3 + 1)-dimensional hyperbolic nonlinear Schrödinger equation (HNLS) is used as a model for different physical phenomena such as the propagation of electromagnetic fields, the dynamics of optical soliton promulgation, and the evolution of the water wave surface. In this paper, new and different exact solutions for the (3 + 1)-dimensional HNLS equation is emerged by using two powerful methods named the Riccati equation method and the F-expansion principle. The behaviors of resulting solutions are different and expressed by dark, bright, singular, and periodic solutions. The physical explanations for the obtained solutions are examined by a graphical representation in 3d profile plots.

Keywords Travelling wave solutions · Solitons · Hyperbolic non-linear Schrödinger equations · Riccati equation method · F-expansion principle

1 Introduction

Many modern phenomena in physics, biology, chemistry, engineering, plasma, optics, and many others may be modeled with nonlinear evolution equations (NLEE's). A better understanding of these phenomena is the focus and interest of many researchers in this field, and therefore great interest was in obtaining exact solutions for these models. Inspired by the last observation, many analytical and numerical methods have emerged for

✉ Mohamed R. Ali
mohamed.reda@bhit.bu.edu.eg

Mahmoud A. Khattab
mahmoud.khatab@bhit.bu.edu.eg

S. M. Mabrouk
s.mabrouk@zu.edu.eg

[1] Basic Engineering Science Department, Benha Faculty of Engineering, Benha University, Benha, Egypt

[2] Faculty of Engineering and Technology, Future University in Egypt, New Cairo 11835, Egypt

[3] Department of Physics and Engineering Mathematics, Faculty of Engineering, Zagazig University, Zagazig, Egypt

this, despite this, there is no specific method to obtain exact solutions for all models, as the method that is effective in one model is ineffective in another one, these methods including inverse scattering transform method (Ali et al. 2022; Ali et al. 2023), Darboux transformation (Monisha et al. 2022), Bäcklund transformation (Dong et al. 2022), the hyperbola function method (Bai 2001), the sine–Gordon expansion method (Kundu, et al. 2021), the improved Bernoulli sub-equation function method (Islam and Ali Akbar 2020; Islam et al. 2020), the lumped Galerkin method (Esen 2005), the Jacobi elliptic function method (Tian 2017), sine–cosine method (Wazwaz 2006), tanh-sech method (Seadawy 2014; Aly 2014), extended tanh-coth method (Bekir 2009), Lie group symmetry analysis method (Sadat et al. 2021; Dig Vijay Tanwar 2022), the first integral method (El-Ganaini 2013), modified extended mapping method (Bai et al. 2011), homogeneous balance method (Aly 2016; Zayed and Arnous 2012), the exp-function method (Remarks and on Exp-Function Method and Its Applications-A Supplement 2013), modified simple-equation method (Irshad et al. 2017; Ali et al. 2017), Kudryashov method (Aly 2017a; Kumar et al. 2018; Barman, et al. 2021), F-expansion method (Zhou et al. 2003; Yıldırım et al. 2020; Yıldırım and Mirzazadeh 2020), Riccati equation method (Bekir 2009; Yıldırım et al. 2020; Yıldırım and Mirzazadeh 2020; Huang and Zhang 2005; Song et al. 2007; El-Ganaini and Kumar 2020) and many others.

The nonlinear Schrödinger equation (NLS) is one of the most important NLEE's which shows its importance in many applications such as optical fibers communications, Heisenberg spin chain, plasma, applied and theoretical physics and many others (El-Ganaini 2013; Li 2007; Gorza 2008; Efremidis et al. 2009; Saha et al. 2009; Ai-Lin and Ji 2010; Triki and Biswas 2011; Guo and Lin 2012; Triki et al. 2012; Ahmed et al. 2014; Zayed 2016; Arshad et al. 2017; Aly 2017b; Mohamed 2018; Seadawy et al. 2018; Islam et al. 2022a, 2022b, 2022c, 2022d, 2022e, 2023; Akbar et al. 2023; Abdullah et al. 2023). The NLS equation takes many forms, among them is the hyperbolic nonlinear Schrödinger (HNLS) equation which the $(2+1)$-dimensional form is given by

$$i\psi_y + \frac{1}{2}\left(\psi_{xx} - \psi_{tt}\right) + \psi^2\psi^* = 0, \tag{1}$$

where $\psi = \psi(x, y, t)$ denote a complex field, x and y are the spatial coordinates and t is the temporal variable. Equation (1) is used in two important applications, the first one is the description of the dynamics of optical soliton promulgation in monomode optical fibers, while the second one is the description of the evolution of the elevation of water wave surface for slowly modulated wave trains in deep water in hydrodynamics. The exact travelling wave solutions for Eq. (1) have been obtained in different contexts by using different powerful methods (Li 2007; Efremidis et al. 2009; Ai-Lin and Ji 2010; Guo and Lin 2012; Ahmed et al. 2014; Zayed 2016; Mohamed 2018; Islam et al. 2022a, 2022b, 2022c, 2023).

In the present article, we aim to study the extension of the HNLS equation in three dimensions namely $(3+1)$-dimensional hyperbolic non-linear Schrödinger (HNLS) equation which is given by Abdullah et al. (2023)

$$i\psi_y + \frac{1}{2}\left(\psi_{xx} + \psi_{zz} - \psi_{tt}\right) + \psi^2\psi^* = 0, \tag{2}$$

where $\psi = \psi(x, y, z, t)$ is a complex- valued function, ψ^* is the complex conjugate of ψ, $i = \sqrt{-1}$, while x, y and z are the spatial variables and t stands for the temporal variable. In the study, we will use two powerful methods, namely Riccati equation technique and F-expansion principle (Yıldırım and Mirzazadeh 2020) to obtain different travelling wave solutions Eq. (2). It is worth noting that, the HNLS (2) has appeared in many different applications, such that, it is used as a model for different physical phenomena such as the propagation of electromagnetic fields, the dynamics of optical soliton promulgation and the evolution of the water wave surface. In the literature, the exact travelling wave solutions for Eq. (2) have been also obtained, such that, in Abdullah et al. (2023) a set of different exact solutions namely, bright, dark, complex, singular and periodic solutions have been obtained.

The rest of the present article is organized as follows: in Sect. 2, a quick review of the Riccati equation technique and its steps followed by different exact travelling wave solutions to Eq. (2) will be illustrated. The instructions of the F-expansion principle and the usage of it as an application to Eq. (2) are shown in Sect. 3. In Sect. 4, the physical explanation of the obtained solutions is examined with 3d profile plots representation. The conclusion of our work is summarized in Sect. 5.

2 Riccati equation technique

In this section, we briefly review the main steps that we take to obtain exact travelling wave solutions for any NLEE in general and apply them to HNLS Eq. (2).

2.1 A quick review of Riccati equation technique

Consider a general governing model given as a NLEE in two independent variables x and t (which may be extended to more than two independent variables) is written as

$$P\left(\psi, \psi_t, \psi_x, \psi_{tt}, \psi_{xx}, \dots\right) = 0, \tag{3}$$

where P and ψ are unknown's functions that have linear and nonlinear derivatives of higher order. The main procedures of the standard Riccati equation technique can be summarized as follows.

Step (1): The given nonlinear evolution Eq. (3) is transformed into an equation of a single variable by using the following travelling wave transformation

$$\psi(x,t) = U(\xi),\ \xi = kx + wt. \tag{4}$$

where k is the wavelength and w is the frequency. Through Eq. (4), the NLEE (3) is transformed into a nonlinear ordinary differential equation (NODE)

$$F\left(U, U', U'', U''', \dots\right) = 0, \tag{5}$$

Step (2): Suppose that the solution structure of Eq. (5) can be written as

$$U(\xi) = \sum_{j=0}^{N} A_j \phi^j(\xi), \tag{6}$$

where A_j are real constants, N is a positive integer which results via the balancing principle with higher order non-linear and linear terms in Eq. (5) and $\phi(\xi)$ satisfies the Riccati equation in the form

$$\phi\prime(\xi) = a\phi^2(\xi) + b\phi + c, \tag{7}$$

where $a \neq 0, b$ and c will be determined.

Step (3): Inserting Eq. (6) together with Eq. (7) into Eq. (5) and equating the coefficients of ϕ^j, $j = 0, 1, \dots, N$ to zero, one can obtain a system of equations in the series of unknowns parameters k, w, A_i, a, b and c which can be solved via *MATHEMATICA* or *MAPLE* software.

Step (4): Putting the obtained values of parameters from the procedure (3) into Eq. (5) through which the travelling wave solution can also be obtained using Eq. (4).

We note that the Riccati Eq. (7) is a non-linear first order ODE that can be solved via the separation of variables. Suppose that

$$\beta = b^2 - 4ac, \tag{8}$$

Then depending on β, the solutions of Eq. (7) can be written as follows.

Case (1): $\beta > 0$,

Case (1–1): $\left|2a\phi + b/\sqrt{\beta}\right| > 1$,

$$\phi(\xi) = -\frac{b}{2a} - \frac{\sqrt{\beta}}{2a}\tanh\left(\frac{\sqrt{\beta}}{2}\xi + \xi_0\right), \tag{9}$$

Case (1–2): $\left|2a\phi + b/\sqrt{\beta}\right| < 1$,

$$\phi(\xi) = -\frac{b}{2a} - \frac{\sqrt{\beta}}{2a}\coth\left(\frac{\sqrt{\beta}}{2}\xi + \xi_0\right), \tag{10}$$

Case (2): $\beta < 0$,

$$\phi(\xi) = -\frac{b}{2a} + \frac{\sqrt{-\beta}}{2a}\tan\left(\frac{\sqrt{-\beta}}{2}\xi + \xi_0\right), \tag{11}$$

$$\phi(\xi) = -\frac{b}{2a} - \frac{\sqrt{-\beta}}{2a}\cot\left(\frac{\sqrt{-\beta}}{2}\xi + \xi_0\right). \tag{12}$$

Case (3): $\beta = 0$,

$$\phi(\xi) = -\frac{b}{2a} - \frac{1}{a\xi + \xi_0}, \tag{13}$$

Where ξ_0 is the integration arbitrary constant.

2.2 Travelling wave solutions via Riccati equation technique

In this subsection, we obtain different soliton solutions for HNLS Eq. (2) by considering the Riccati equation method. First, apply the following travelling wave transformation to HNLS Eq. (2)

$$\psi(x, y, z, t) = U(\xi)e^{i\varphi(x,y,z,t)}, \tag{14}$$

where

$$\xi(x,y,z,t) = \eta\left(B_1 x + B_2 y + B_3 z - \rho t\right), \tag{15}$$

$$\varphi(x,y,z,t) = -k_1 x - k_2 y - k_3 z + wt, \tag{16}$$

From Eqs. (2) and (14), we obtain the following derivatives

$$\begin{aligned} i\psi_y &= \left(k_2 U + i\eta B_2 U'\right)e^{i\varphi},\\ \frac{1}{2}\psi_{xx} &= \left(-\frac{1}{2}k_1^2 U - ik_1\eta B_1 U' + \frac{1}{2}\eta^2 B_1^2 U''\right)e^{i\varphi},\\ \frac{1}{2}\psi_{zz} &= \left(-\frac{1}{2}k_3^2 U - ik_3\eta B_3 U' + \frac{1}{2}\eta^2 B_3^2 U''\right)e^{i\varphi},\\ \frac{1}{2}\psi_{tt} &= \left(\frac{1}{2}\eta^2 - \frac{1}{2}w^2 U - iw\eta\rho U'\right)e^{i\varphi},\\ \psi^2\psi^* &= U^3 e^{i\varphi}. \end{aligned} \tag{17}$$

Inserting the obtained results from Eq. (17) into Eq. (2), then Eq. (2) is then converted to non-linear ODE which can be written in the following real and imaginary parts.

The real part:

$$\left(\eta^2 B_1^2 + \eta^2 B_3^2 - \eta^2\rho^2\right)U'' + \left(2k_2 - k_1^2 - k_3^2 + w^2\right)U + 2U^3 = 0, \tag{18}$$

The imaginary part:

$$\rho w - k_1 B_1 - k_3 B_3 + B_2 = 0, \tag{19}$$

Using Eq. (18) and assuming that it's solution can be written as

$$U(\xi) = \sum_{j=0}^{N} A_j \phi^j(\xi), \tag{20}$$

where A_j are real constant will be determined and $\phi(\xi)$ satisfies the Riccati Eq. (7).

According to the balancing principle and from Eq. (18) we conclude that $N = 1$ and hence the series in Eq. (20) can be expanded as

$$U = A_0 + A_1\phi, \tag{21}$$

From Eq. (21) together with Eq. (7) we obtain the following formulas for U', U'' and U^3

$$\begin{aligned} U'' &= 2a^2 A_1\phi^3 + 3abA_1\phi^2 + \left(2acA_1 + b^2 A_1\right)\phi + bcA_1,\\ U^3 &= A_1^3\phi^3 + 3A_0A_1^2\phi^2 + 3A_0^2A_1\phi + A_0^3, \end{aligned} \tag{22}$$

Inserting the obtained results from Eq. (22) into Eq. (18) and equating the coefficients of different exponents of ϕ to zero, we obtain the following system of equations

$$\begin{aligned} &\left(2a^2A_1\right)\left(\eta^2B_1^2 + \eta^2B_3^2 - \eta^2\rho^2\right) + 2A_1^3 = 0,\\ &\left(3abA_1\right)\left(\eta^2B_1^2 + \eta^2B_3^2 - \eta^2\rho^2\right) + 6A_0A_1^2 = 0,\\ &\left(2acA_1 + b^2A_1\right)\left(\eta^2B_1^2 + \eta^2B_3^2 - \eta^2\rho^2\right) + A_1\left(2k_2 - k_1^2 - k_3^2 + w^2\right) + 6A_0^2A_1 = 0,\\ &\left(bcA_1\right)\left(\eta^2B_1^2 + \eta^2B_3^2 - \eta^2\rho^2\right) + A_0\left(2k_2 - k_1^2 - k_3^2 + w^2\right) + 2A_0^3 = 0. \end{aligned} \tag{23}$$

Using ***MAPEL*** software to solve the system in Eq. (23), we obtain the following important results for a, b and c

$$a = \pm \frac{A_1}{\eta\sqrt{\rho^2 - B_1^2 - B_3^2}}, \tag{24}$$

$$b = \pm \frac{2A_0}{\eta\sqrt{\rho^2 - B_1^2 - B_3^2}}, \tag{25}$$

$$c = \pm \frac{2A_0^2 + w^2 - k_1^2 - k_3^2 + 2k_2}{2A_1\eta\sqrt{\rho^2 - B_1^2 - B_3^2}}. \tag{26}$$

From Eqs. (24–26) and using Eq. (8) we have

$$\beta = -\frac{2\left(w^2 - k_1^2 - k_3^2 + 2k_2\right)}{\left(\rho^2 - B_1^2 - B_3^2\right)\eta^2}. \tag{27}$$

Inserting Eqs. (24–27) into Eqs. (9–13) and by using Eqs. (21, 14), the travelling wave solutions for HNLS Eq. (2) will be obtained as follows.

Case (1): when $\beta > 0$ (see Eq. (27)), we obtain the following travelling wave solutions.

Case (1–1): Dark optical soliton solution

$$\psi_1(x,y,z,t) = \pm\sqrt{\frac{k_1^2 + k_3^2 - w^2 - 2k_2}{2}}\tanh\left(\left(\rho t - B_1 x - B_2 y - B_3 z\right)\sqrt{\frac{k_1^2 + k_3^2 - w^2 - 2k_2}{2\left(\rho^2 - B_1^2 - B_3^2\right)}} + \xi_0\right)e^{i\left(wt - k_1 x - k_2 y - k_3 z\right)}, \tag{28}$$

Case (1–2): Singular soliton solution

$$\psi_2(x,y,z,t) = \pm\sqrt{\frac{k_1^2 + k_3^2 - w^2 - 2k_2}{2}}\coth\left(\left(\rho t - B_1 x - B_2 y - B_3 z\right)\sqrt{\frac{k_1^2 + k_3^2 - w^2 - 2k_2}{2\left(\rho^2 - B_1^2 - B_3^2\right)}} + \xi_0\right)e^{i\left(wt - k_1 x - k_2 y - k_3 z\right)}, \tag{29}$$

Case (2): when $\beta < 0$ (see Eq. (27)), we obtain the following singular periodic soliton solutions

$$\psi_3(x,y,z,t) = \pm\sqrt{\frac{-k_1^2 - k_3^2 + w^2 + 2k_2}{2}}\tan\left(\left(\rho t - B_1 x - B_2 y - B_3 z\right)\sqrt{\frac{-k_1^2 - k_3^2 + w^2 + 2k_2}{2\left(\rho^2 - B_1^2 - B_3^2\right)}} + \xi_0\right)e^{i\left(wt - k_1 x - k_2 y - k_3 z\right)} \tag{30}$$

$$\psi_4(x,y,z,t) = \pm\sqrt{\frac{-k_1^2 - k_3^2 + w^2 + 2k_2}{2}}\cot\left(\left(\rho t - B_1 x - B_2 y - B_3 z\right)\sqrt{\frac{-k_1^2 - k_3^2 + w^2 + 2k_2}{2\left(\rho^2 - B_1^2 - B_3^2\right)}} + \xi_0\right)e^{i\left(wt - k_1 x - k_2 y - k_3 z\right)} \tag{31}$$

where ρ is defined in Eq. (19).

3 F-expansion principle

In this section, similarly, as in the previous sections, we will initially review the steps followed in.

F-expansion method to obtain exact soliton solutions of NLEE's and follow them to obtain exact soliton solutions of HNLS Eq. (2).

3.1 A quick review of F-expansion principle

Consider a general NLEE in two independent variables x and t (which may be extended to more than two independent variables) is written as

$$P\left(\psi, \psi_t, \psi_x, \psi_{tt}, \psi_{xx}, \dots\right) = 0 \tag{32}$$

where P and ψ are unknowns' functions that have linear and nonlinear derivatives of higher order. The main procedures of the standard F-expansion principle can be summarized as follows.

Step 1: The given nonlinear evolution Eq. (32) is transformed into an equation of a single variable by using the following travelling wave transformation

$$\psi(x,t) = U(\xi), \xi = kx + wt \tag{33}$$

where k is the wavelength and w is the frequency. Through Eq. (33), the NLEE (32) is transformed to a nonlinear ordinary differential equation (NODE)

$$F\left(U, U\prime, U'', U''', \dots\right) = 0, \tag{34}$$

Step 2: Suppose that the solution structure of Eq. (34) can be written as

$$U(\xi) = \sum_{j=0}^{N} s_j F^j(\xi), \tag{35}$$

where s_j are real constants, N is positive integer which result via the balancing principle with higher order non-linear and linear terms in Eq. (34) and $\phi(\xi)$ satisfies the following first order ODE

$$F\prime(\xi) = \sqrt{PF^4(\xi) + QF^2 + R.} \tag{36}$$

where $T \neq 0, Q$ and R will be determined.

Step 3: Inserting Eq. (35) together with Eq. (36) into Eq. (34) and equating the coefficients of.

F^j, $j = 0, 1, \dots, N$ to zero, one can obtain a system of equations in the unknowns parameters k, w, s_i, P, Q and R which can be solved via *MATHEMATICA* or *MAPLE* software.

Step 4: Putting the obtained values of parameters from procedure (3) into Eq. (34) through which the travelling wave solution can also be obtained using Eq. (33).

We note that, in Table 1, the solutions of Eq. (36) are illustrated in terms of Jacobian elliptic functions, for more details we recommended (Akram et al. 2023).

Table 1 Solutions of Eq. (36) in terms of Jacobian elliptic functions

Conditions on the parameters			The general solution	The solution when $m \to 1$	The solution when $m \to 0$
P	Q	R			
m^2	$-1-m^2$	1	$F(\xi)=sn(\xi)$	$F(\xi)=tanh(\xi)$	$F(\xi)=sin(\xi)$
$-m^2$	$2m^2-1$	$1-m^2$	$F(\xi)=cn(\xi)$	$F(\xi)=sech(\xi)$	$F(\xi)=cos(\xi)$
-1	$2-m^2$	m^2-1	$F(\xi)=dn(\xi)$	$F(\xi)=sech(\xi)$	$F(\xi)=1$
1	$-1-m^2$	m^2	$F(\xi)=ns(\xi)$	$F(\xi)=\coth(\xi)$	$F(\xi)=\csc(\xi)$
$1-m^2$	$2m^2-1$	$-m^2$	$F(\xi)=nc(\xi)$	$F(\xi)=\cosh(\xi)$	$F(\xi)=\sec(\xi)$
m^2-1	$2-m^2$	-1	$F(\xi)=nd(\xi)$	$F(\xi)=\cosh(\xi)$	$F(\xi)=1$
$1-m^2$	$2-m^2$	1	$F(\xi)=sc(\xi)$	$F(\xi)=\sinh(\xi)$	$F(\xi)=\tan(\xi)$
m^4-m^2	$2m^2-1$	1	$F(\xi)=sd(\xi)$	$F(\xi)=\sinh(\xi)$	$F(\xi)=\sin(\xi)$
1	$2-m^2$	$1-m^2$	$F(\xi)=cs(\xi)$	$F(\xi)=\mathrm{csch}(\xi)$	$F(\xi)=\cot(\xi)$
1	$2m^2-1$	m^4-m^2	$F(\xi)=ds(\xi)$	$F(\xi)=\mathrm{csch}(\xi)$	$F(\xi)=\csc(\xi)$
0.25	$0.5-m^2$	0.25	$F(\xi)=ns(\xi)\pm cs(\xi)$	$F(\xi)=\coth(\xi)\pm\mathrm{csch}(\xi)$	$F(\xi)=\csc(\xi)\pm\cot(\xi)$
$(1-m^2)/4$	$(1+m^2)/2$	$(1-m^2)/4$	$F(\xi)=nc(\xi)\pm sc(\xi)$	$F(\xi)=\cosh(\xi)\pm\sinh(\xi)$	$F(\xi)=\sec(\xi)\pm\tan(\xi)$
0.25	$(m^2/2)-1$	$m^2/4$	$F(\xi)=ns(\xi)\pm ds(\xi)$	$F(\xi)=\coth(\xi)\pm\mathrm{csch}(\xi)$	$F(\xi)=\csc(\xi)\pm\csc(\xi)$
$m^2/4$	$(m^2/2)-1$	$m^2/4$	$F(\xi)=sn(\xi)\pm cn(\xi)$	$F(\xi)=\tanh(\xi)\pm sech(\xi)$	$F(\xi)=\sin(\xi)\pm cos(\xi)$

3.2 Travelling wave solutions via F-expansion principle

In this subsection, by following the steps mentioned in the F-expansion method in the previous section, we begin by applying the travelling wave transformation in Eq. (14) to HNLS Eq. (2), then Eq. (2) is converted to a nonlinear ODE which it's real and imaginary parts are written in Eqs. (18) and (19), respectively. Using Eq. (18) and assuming that it's solution can be written as

$$U(\xi) = \sum_{j=0}^{N} s_j F^j(\xi), \tag{37}$$

where s_j are real constant will be determined and $F(\xi)$ satisfies Eq. (36).

According to the balancing principle, then from Eq. (18) we conclude that $N = 1$ and hence the series in Eq. (37) can be expanded as

$$U = s_0 + s_1 F, \tag{38}$$

From Eq. (38) together with Eq. (36) we obtain the following formulas

$$\begin{aligned} U'' &= 2s_1 PF^3 + s_1 QF, \\ U^3 &= s_1^3 F^3 + 3s_0 s_1^2 F^2 + 3s_0^2 s_1 F + s_0^3, \end{aligned} \tag{39}$$

Using Eq. (39) together with Eq. (18) and equating the different exponent of $F^j, j = 0, 1, 2, 3$ to zero, we obtain the following system of equations

$$\begin{aligned} &\left(s_1 Q\right)\left(\eta^2 B_1^2 + \eta^2 B_3^2 - \eta^2 \rho^2\right) + s_1\left(2k_2 - k_1^2 - k_3^2 + w^2\right) + 6s_0^2 s_1 = 0, \\ &\left(2s_1 P\right)\left(\eta^2 B_1^2 + \eta^2 B_3^2 - \eta^2 \rho^2\right) + 2s_1^3 = 0, \\ &s_0\left(2k_2 - k_1^2 - k_3^2 + w^2\right) + 2s_0^3 = 0, 6s_0 s_1^2 = 0. \end{aligned} \tag{40}$$

The solution of system (40) using ***MAPLE*** software gives.

$$s_0 = 0, s_1 = \pm\sqrt{\frac{P\left(2k_2 - k_1^2 - k_3^2 + w^2\right)}{Q}}, \eta = \pm\sqrt{\frac{k_1^2 + k_3^2 - w^2 - 2k_2}{Q\left(B_1^2 + B_3^2 - \rho^2\right)}}. \tag{41}$$

Inserting the obtained values from Eq. (41) into Eq. (38) and using Table 1, the travelling wave solutions for HNLS in Eq. (2) are obtained as follows.

Case (1): Dark-soliton solution

$$\psi_5 = \pm\sqrt{k_1^2 + k_3^2 - 2k_2 - w^2} tanh\left(\sqrt{\frac{k_1^2 + k_3^2 - w^2 - 2k_2}{\rho^2 - B_1^2 - B_3^2}}\left(B_1 x + B_2 y + B_3 z - \rho t\right)\right) e^{i\left(wt - k_1 x - k_2 y - k_3 z\right)}, \tag{42}$$

Case (2): Bright-soliton solution

$$\psi_6 = \pm\sqrt{k_1^2 + k_3^2 - 2k_2 - w^2} sech\left(\sqrt{\frac{k_1^2 + k_3^2 - w^2 - 2k_2}{B_1^2 + B_3^2 - \rho^2}}\left(B_1 x + B_2 y + B_3 z - \rho t\right)\right) e^{i\left(wt - k_1 x - k_2 y - k_3 z\right)}, \tag{43}$$

Case (3): Singular-soliton solutions

$$\psi_7 = \pm\sqrt{\frac{k_1^2 + k_3^2 - 2k_2 - w^2}{2}} coth\left(\sqrt{\frac{k_1^2 + k_3^2 - w^2 - 2k_2}{2\left(\rho^2 - B_1^2 - B_3^2\right)}}\left(B_1x + B_2y + B_3z - \rho t\right)\right)e^{i\left(wt - k_1x - k_2y - k_3z\right)}, \tag{44}$$

$$\psi_8 = \pm\sqrt{2k_2 - k_1^2 - k_3^2 + w^2} csch\left(\sqrt{\frac{k_1^2 + k_3^2 - w^2 - 2k_2}{B_1^2 + B_3^2 - \rho^2}}\left(B_1x + B_2y + B_3z - \rho t\right)\right)e^{i\left(wt - k_1x - k_2y - k_3z\right)}. \tag{45}$$

Case (4): Singular-periodic soliton solutions

$$\psi_9 = \pm\sqrt{\frac{2k_2 - k_1^2 - k_3^2 + w^2}{2}} tan\left(\sqrt{\frac{k_1^2 + k_3^2 - w^2 - 2k_2}{2\left(B_1^2 + B_3^2 - \rho^2\right)}}\left(B_1x + B_2y + B_3z - \rho t\right)\right)e^{i\left(wt - k_1x - k_2y - k_3z\right)}, \tag{46}$$

$$\psi_{10} = \pm\sqrt{\frac{2k_2 - k_1^2 - k_3^2 + w^2}{2}} cot\left(\sqrt{\frac{k_1^2 + k_3^2 - w^2 - 2k_2}{2\left(B_1^2 + B_3^2 - \rho^2\right)}}\left(B_1x + B_2y + B_3z - \rho t\right)\right)e^{i\left(wt - k_1x - k_2y - k_3z\right)}, \tag{47}$$

$$\psi_{11} = \pm\sqrt{k_1^2 + k_3^2 - 2k_2 - w^2} csc\left(\sqrt{\frac{k_1^2 + k_3^2 - w^2 - 2k_2}{\rho^2 - B_1^2 - B_3^2}}\left(B_1x + B_2y + B_3z - \rho t\right)\right)e^{i\left(wt - k_1x - k_2y - k_3z\right)}, \tag{48}$$

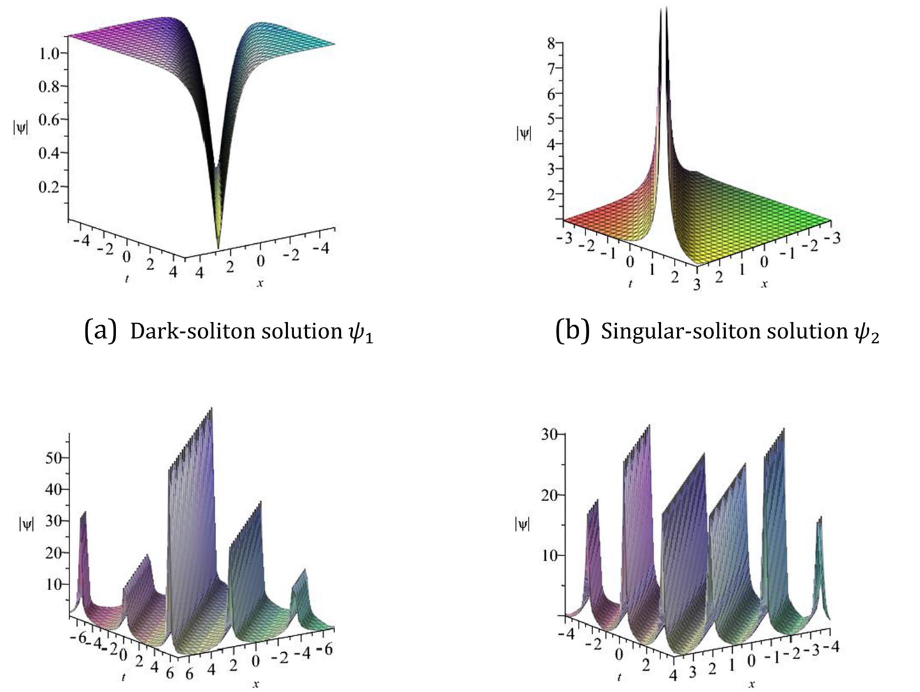
$$\psi_{12} = \pm\sqrt{k_1^2 + k_3^2 - 2k_2 - w^2} sec\left(\sqrt{\frac{k_1^2 + k_3^2 - w^2 - 2k_2}{\rho^2 - B_1^2 - B_3^2}}\left(B_1x + B_2y + B_3z - \rho t\right)\right)e^{i\left(wt - k_1x - k_2y - k_3z\right)}. \tag{49}$$

Case (5): Cluster-singular soliton solution

$$\begin{aligned}\psi_{13} = \pm\sqrt{\frac{k_1^2 + k_3^2 - 2k_2 - w^2}{2}}&\left[coth\left(\sqrt{\frac{2(k_1^2 + k_3^2 - w^2 - 2k_2)}{\rho^2 - B_1^2 - B_3^2}}\left(B_1x + B_2y + B_3z - \rho t\right)\right)\right.\\ &\left.\pm csch\left(\sqrt{\frac{2(k_1^2 + k_3^2 - w^2 - 2k_2)}{\rho^2 - B_1^2 - B_3^2}}\left(B_1x + B_2y + B_3z - \rho t\right)\right)\right]e^{i\left(wt - k_1x - k_2y - k_3z\right)}.\end{aligned} \tag{50}$$

Case (6): Dark-bright soliton solution

$$\begin{aligned}\psi_{14} = \pm\sqrt{\frac{k_1^2 + k_3^2 - 2k_2 - w^2}{2}}&\left[tanh\left(\sqrt{\frac{2(k_1^2 + k_3^2 - w^2 - 2k_2)}{\rho^2 - B_1^2 - B_3^2}}\left(B_1x + B_2y + B_3z - \rho t\right)\right)\right.\\ &\left.\pm sech\left(\sqrt{\frac{2(k_1^2 + k_3^2 - w^2 - 2k_2)}{\rho^2 - B_1^2 - B_3^2}}\left(B_1x + B_2y + B_3z - \rho t\right)\right)\right]e^{i\left(wt - k_1x - k_2y - k_3z\right)}.\end{aligned} \tag{51}$$



(a) Dark-soliton solution ψ_1

(b) Singular-soliton solution ψ_2

(c) Singular periodic soliton solution ψ_3

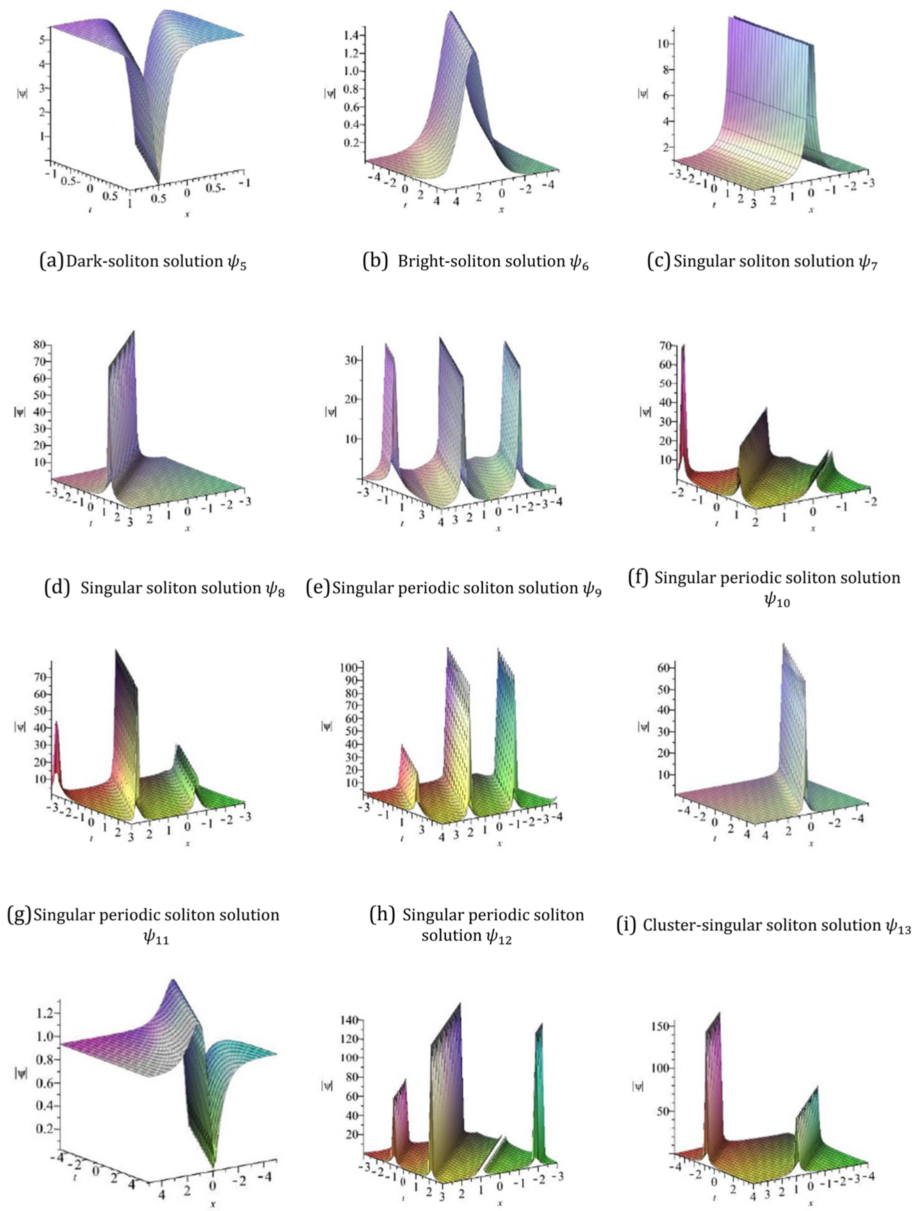
(d) Singular periodic soliton solution ψ_4

Fig. 1 The solution structure for soliton solutions obtained by Riccati equation technique

Case (7): Cluster-singular periodic soliton solutions

$$\psi_{15} = \pm\sqrt{\frac{2k_2 - k_1^2 - k_3^2 + w^2}{2}}\left[csc\left(\sqrt{\frac{2(k_1^2 + k_3^2 - w^2 - 2k_2)}{B_1^2 + B_3^2 - \rho^2}}\left(B_1x + B_2y + B_3z - \rho t\right)\right) \pm cot\left(\sqrt{\frac{2(k_1^2 + k_3^2 - w^2 - 2k_2)}{B_1^2 + B_3^2 - \rho^2}}\left(B_1x + B_2y + B_3z - \rho t\right)\right)\right]e^{i\left(wt - k_1x - k_2y - k_3z\right)}. \tag{52}$$

$$\psi_{16} = \pm\sqrt{\frac{2k_2 - k_1^2 - k_3^2 + w^2}{2}}\left[sec\left(\sqrt{\frac{2(k_1^2 + k_3^2 - w^2 - 2k_2)}{B_1^2 + B_3^2 - \rho^2}}\left(B_1x + B_2y + B_3z - \rho t\right)\right) \pm tan\left(\sqrt{\frac{2(k_1^2 + k_3^2 - w^2 - 2k_2)}{B_1^2 + B_3^2 - \rho^2}}\left(B_1x + B_2y + B_3z - \rho t\right)\right)\right]e^{i\left(wt - k_1x - k_2y - k_3z\right)}. \tag{53}$$



(a) Dark-soliton solution ψ_5 (b) Bright-soliton solution ψ_6 (c) Singular soliton solution ψ_7

(d) Singular soliton solution ψ_8 (e) Singular periodic soliton solution ψ_9 (f) Singular periodic soliton solution ψ_{10}

(g) Singular periodic soliton solution ψ_{11} (h) Singular periodic soliton solution ψ_{12} (i) Cluster-singular soliton solution ψ_{13}

(j) Dark-Bright soliton solution ψ_{14} (k) Cluster-singular periodic soliton solution ψ_{15} (l) Cluster-singular periodic soliton solution ψ_{16}

Fig. 2 The solution structure for the obtained soliton solutions by F-expansion principle where ρ is defined in Eq. (19).

4 Results, physical examination, and discussion

In this section, we will illustrate graphically some solutions for the obtained general ones in the previous section. These solutions are coming from giving the arbitrary parameters $k_1, k_2, k_3, B_1, B_2, B_3$ and w a special value.

Figure 1 shows the solution structure for soliton solutions was obtained by Riccati equation method. Figure 1a shows the dark-soliton solution ψ_1 with parameters values

$B_1 = 0.8$, $B_2 = 0.6$, $B_3 = 1$, $k_1 = 0.5$, $k_2 = k_3$=1, w =1.3, $\eta = 1$, $\xi_0 = 0$, y = −2, z = 1.

Figure (1-b) illustrates the singular soliton solution ψ_2 with

$B_1 = 2$, $B_2 = 1$, $B_3 = 2$, $k_1 = 0.5$, $k_2 = k_3 = 1$, w = 1, $\eta = 1$, $\xi_0 = 0$, y = 1, z = −2.

In Fig. 1c, the singular periodic soliton solution ψ_3 is shown with parameters values

$B_1 = 1$, $B_2 = 2$, B_3=2, $k_1 = 1$, $k_2 = -0.7$, $k_3 = 1$, $w = 1$, $\eta = 1$, $\xi_0 = 0$, $y = 2$, $z = -4$,

While in Fig. 1d, the singular periodic soliton solution ψ_4 is shown with parameters values

$$B_1 = B_2 = B_3 = -1, k_1 = 1, k_2 = -0.8, k_3 = 1.4, w = 1.4, \eta = -1, \xi_0 = -1, y = -\sqrt{2}, z = \sqrt{2},$$

Figure 2 presents the graphical representation for different solutions of HNLS equation using F-expansion method.

In Fig. 2a, the dark-soliton solution ψ_5 is shown with

$$B_1 = B_2 = 2, B_3 = -2, k_1 = 1.1, k_2 = 9, k_3 = 2.3, w = -4.4, y = 1, z = 1,$$

While in Fig. 2b, the bright-soliton solution ψ_6 is illustrated with

$$B_1 = 1.1, B_2 = B_3 = 1.2, k_1 = 0.5, k_2 = -1, k_3 = w = 1, y = -2, z = 2.$$

Figure 2c shows the singular soliton solution ψ_7 with

$$B_1 = -2, B_2 = 1, B_3 = 2, k_1 = 0.5, k_2 = k_3 = w = 1, y = 3, z = -1.5.$$

In Fig. 2d, the singular-soliton solution ψ_8 is shown with

$$B_1 = 1, B_2 = B_3 = 2, k_1 = k_3 = 1, k_2 = -0.7, w = 1, y = 2.2, z = -3.$$

Figure 2e–h shows the periodic singular soliton solutions $\psi_9, \psi_{10}, \psi_{11}$ andψ_{12}, respectively. The parameters values are $B_1 = B_2 = B_3 = K_3 = w = y = z = 1$ in such figures except for k_1 andk_2, such that in Fig. (2-e), $k_1 = 0.5, k_2 = -1$, while in Fig. 2f, $k_1 = 1, k_2 = -0.7$ but in Fig. 2g and h), $k_1 = 0.5, k_2 = 1$.

Figure 2i and j shows the cluster-singular soliton solution ψ_{13} and the dark-bright soliton solution ψ_{14} with parameters values $B_1 = B_2 = B_3 = k_2 = k_3 = w = y = z = 1$ and $k_1 = 0.5$, while in Fig. 2k and l, the Cluster-singular periodic soliton solutions ψ_{15} and ψ_{16} are shown with $B_1 = B_2 = B_3 = k_1 = k_3 = w = y = z = 1, k_2 = -0.7$ and $B_1 = 0.2, B_2 = B_3 = 0.5, k_1 = k_3 = w = y = z = 1, k_2 = -0.5$, respectively.

5 Conclusion

In the present article, we have investigated the travelling wave solutions of the $(3+1)$-dimensional hyperbolic nonlinear Schrödinger equation (HNLS) because of its paramount importance in several applications such that it is used as a model for different physical phenomena such as the propagation of electromagnetic fields, the dynamics of optical soliton promulgation and the evolution of the water wave surface. The study was done from the point of view of two powerful methods, namely, Riccati equation technique and F-expansion principle. Different and new solutions were obtained for that equation and physically examined by graphical representations in 3d plots.

Author contributions MRA, MAK, SMM have proposed the idea. MRA has done the simulations. All authors have contributed to the paper's analysis, discussion, writing, and revision.

Funding Open access funding provided by The Science, Technology & Innovation Funding Authority (STDF) in cooperation with The Egyptian Knowledge Bank (EKB). None.

Declarations

Conflict of interest The authors declare that there are no conflicts of interest related to this article.

Ethical approval The authors would like to clarify that there are no financial/non-financial interests that are directly or indirectly related to the work submitted for publication.

References

Abdullah, F.A., Islam, M.T., Gómez-Aguilar, J.F., Akbar, M.A.: Impressive and innovative soliton shapes for nonlinear Konno-Oono system relating to electromagnetic field. Opt. Quant. Electron. **55**(1), 1–12 (2023)

Ahmed, I., et al.: Exact solution of the (2+ 1)-dimensional hyperbolic nonlinear Schrodinger equation by Adomian decomposition method. Malaya J. Mat **2**, 160–164 (2014)

Ai-Lin, G., Ji, L.: Exact solutions of (2+ 1)-dimensional HNLS equation. Commun. Theor. Phys. **54**, 112–123 (2010)

Akbar, M.A., Abdullah, F.A., Islam, M.T., Al Sharif, M.A., Osman, M.S.: New solutions of the soliton type of shallow water waves and superconductivity models. Results Phys. **44**, 106180 (2023)

Akram, S., Ahmad, J., Sarwar, S., Ali, A.: Dynamics of soliton solutions in optical fibers modelled by perturbed nonlinear Schrödinger equation and stability analysis. Opt. Quant. Electron. **55**(5), 45–59 (2023)

Ali, A., et al.: Soliton solutions of the nonlinear Schrödinger equation with the dual power law nonlinearity and resonant nonlinear Schrödinger equation and their modulation instability analysis. Optik **145**, 79–88 (2017)

Ali, M.R., et al.: Mathematical examination for the energy flow in an inhomogeneous Heisenberg ferromagnetic chain. Optik **271**, 170138 (2022)

Ali, M.R., et al.: Optical soliton solutions for the integrable Lakshmanan-Porsezian-Daniel equation via the inverse scattering transformation method with applications. Optik **272**, 170256 (2023)

Arshad, M., et al.: Bright–dark solitary wave solutions of generalized higher-order nonlinear Schrödinger equation and its applications in optics. J. Electromag. Waves Appl. **31**, 1711–1721 (2017)

Bai, C.: Exact solutions for nonlinear partial differential equation: a new approach. Phys. Lett. A **288**, 191–195 (2001)

Bai, C.-J., et al.: New traveling wave solutions for a class of nonlinear evolution equations. Int. J. Mod. Phys. B **25**, 319–327 (2011)

Barman, H.K., et al.: Solutions to the Konopelchenko-Dubrovsky equation and the Landau-Ginzburg-Higgs equation via the generalized Kudryashov technique. Results Phys. **24**, 104092 (2021)

Bekir, A.: The tanh–coth method combined with the Riccati equation for solving non-linear equation. Chaos, Solitons Fractals **40**, 1467–1474 (2009)

Dong, S., et al.: Bäcklund transformation and multi-soliton solutions for the discrete Korteweg–de Vries equation. Appl. Math. Lett. **125**, 107747 (2022)

Efremidis, N.K., et al.: Exact X-wave solutions of the hyperbolic nonlinear Schrödinger equation with a supporting potential. Phys. Lett. A **373**, 4073–4076 (2009)

El-Ganaini, S.I.A.: The first integral method to the nonlinear schrodinger equations in higher dimensions. Abstr. Appl. Anal. **2013**, 349173 (2013)

El-Ganaini, S., Kumar, H.: A variety of new traveling and localized solitary wave solutions of a nonlinear model describing the nonlinear low- pass electrical transmission lines. Chaos, Solitons Fractals **140**, 110218 (2020)

Esen, A.: A numerical solution of the equal width wave equation by a lumped Galerkin method. Appl. Math. Comput. **168**, 270–282 (2005)

Gorza, S.P.: Marc Haelterman, Ultrafast transverse undulation of self-trapped laser beams. Opt. Express **16**, 16935–16940 (2008)

Guo, A.-L., Lin, Ji.: (2+ 1)-Dimensional analytical solutions of the combining cubic-quintic nonlinear Schrödinger equation. Commun. Theor. Phys. **57**, 21–35 (2012)

Huang, D.-J., Zhang, H.-Q.: Variable-coefficient projective Riccati equation method and its application to a new (2+1)-dimensional simplified generalized Broer-Kaup system. Chaos, Solitons Fractals **23**, 601–607 (2005)

Irshad, A., et al.: A new modification in simple equation method and its applications on nonlinear equations of physical nature. Results Phys. **7**, 4232–4240 (2017)

Islam, Md., et al.: Search for interactions of phenomena described by the coupled Higgs field equation through analytical solutions. Opt. Quant. Electron. **52**, 1–19 (2020)

Islam, M.T., Akbar, M.A., Ahmad, H.: Diverse optical soliton solutions of the fractional coupled (2+ 1)-dimensional nonlinear Schrödinger equations. Opt. Quant. Electron. **54**(2), 45–58 (2022a)

Islam, M.T., Abdullah, F.A., Gómez-Aguilar, J.F.: A variety of solitons and other wave solutions of a nonlinear Schrödinger model relating to ultra-short pulses in optical fibers. Opt. Quant. Electron. **54**(12), 111–124 (2022b)

Islam, M.T., Akbar, M.A., Gómez-Aguilar, J.F., Bonyah, E., Fernandez-Anaya, G.: Assorted soliton structures of solutions for fractional nonlinear Schrodinger types evolution equations. J. Ocean Eng. Sci. **7**(6), 528–535 (2022c)

Islam, M.T., Akbar, M.A., Ahmad, H., Ilhan, O.A., Gepreel, K.A.: Diverse and novel soliton structures of coupled nonlinear Schrödinger type equations through two competent techniques. Mod. Phys. Lett. B **36**(11), 2250004 (2022d)

Islam, M.T., Akter, M.A., Gómez-Aguilar, J.F., Akbar, M.A., Perez-Careta, E.: Novel optical solitons and other wave structures of solutions to the fractional order nonlinear Schrodinger equations. Opt. Quant. Electron. **54**(8), 66–78 (2022e)

Islam, M.E., AliAkbar, M.: Stable wave solutions to the Landau-Ginzburg-Higgs equation and the modified equal width wave equation using the IBSEF method. Arab J. Basic Appl. Sci. **27**, 270–278 (2020)

Islam, M.T., Akter, M.A., Gomez-Aguilar, J.F., Akbar, M.A., & Pérez-Careta, E. Innovative and diverse soliton solutions of the dual core optical fiber nonlinear models via two competent techniques. J. Nonlinear Optic. Phys. Mater., 2350037 (2023)

Kumar, D., et al.: Modified Kudryashov method via new exact solutions for some conformable fractional differential equations arising in mathematical biology. Chin. J. Phys. **56**, 75–85 (2018)

Kundu, P.R., et al.: Linear and nonlinear effects analysis on wave profiles in optics and quantum physics. Results Phys **23**, 103995 (2021)

Li, B.: A generalized sub-equation expansion method and its application to the nonlinear schrödinger equation in inhomogeneous optical fiber media. Int. J. Mod. Phys. C **18**, 1187–1201 (2007)

Osman, M.S.: On multi-soliton solutions for the (2+ 1)-dimensional breaking soliton equation with variable coefficients in a graded-index waveguide. Comput. Math. Appl. **75**, 1–6 (2018)

Monisha, S., et al.: Higher order smooth positon and breather positon solutions of an extended nonlinear Schrödinger equation with the cubic and quartic nonlinearity. Chaos, Solitons Fractals **162**, 112433 (2022)

Sadat, R., et al.: Lie symmetry analysis and invariant solutions of 3D Euler equations for axisymmetric, incompressible, and inviscid flow in the cylindrical coordinates. Adv. Differ. Equ. **2021**, 88–101 (2021)

Saha, M., et al.: Dark optical solitons in power law media with time-dependent coefficients. Phys. Lett. A **373**, 4438–4441 (2009)

Seadawy, A.R.: Stability analysis for Zakharov-Kuznetsov equation of weakly nonlinear ion-acoustic waves in a plasma. Comput. Math. Appl. **67**, 172–180 (2014)

Seadawy, A.R., et al.: Dispersive optical soliton solutions for the hyperbolic and cubic-quintic nonlinear Schrödinger equations via the extended sinh-Gordon equation expansion method. Eur. Phys. J. plus **133**, 105-117 (2018)

Seadawy, A.R.: Stability analysis for two-dimensional ion-acoustic waves in quantum plasmas. Phys. Plasmas **21**, 052107 (2014)

Seadawy, A.R.: Stability analysis solutions for nonlinear three-dimensional modified Korteweg–de Vries–Zakharov–Kuznetsov equation in a magnetized electron–positron plasma. Physica A **455**, 44–51 (2016)

Seadawy, A.R.: Modulation instability analysis for the generalized derivative higher order nonlinear Schrödinger equation and its the bright and dark soliton solutions. J. Electromagn. Waves Appl. **31**, 1353–1362 (2017a)

Seadawy, A.R., Lu, D.: Bright and dark solitary wave soliton solutions for the generalized higher order nonlinear Schrödinger equation and its stability. Results Phys. **7**, 43–48 (2017b)

Song, L.-N., et al.: A new extended Riccati equation rational expansion method and its application. Chaos, Solitons Fractals **31**, 548–556 (2007)

Some remarks on exp-function method and its applications-a supplement, Commun. Theor. Phys., 60, 521 (2013).

Tanwar, D.V.: Lie symmetry reductions and generalized exact solutions of Date–Jimbo–Kashiwara–Miwa equation. Chaos, Solitons Fractals **162**, 112414 (2022)

Tian, S.-F.: Initial–boundary value problems for the general coupled nonlinear Schrödinger equation on the interval via the Fokas method. J. Differ. Equ. **262**, 506–558 (2017)

Triki, H., Biswas, A.: Dark solitons for a generalized nonlinear Schrödinger equation with parabolic law and dual-power law nonlinearities. Math. Methods Appl. Sci. **34**, 958–962 (2011)

Triki, H., et al.: Bright and dark solitons for the resonant nonlinear Schrödinger's equation with time-dependent coefficients. Opt. Laser Technol. **44**, 2223–2231 (2012)

Wazwaz, A.-M.: The tanh and the sine–cosine methods for a reliable treatment of the modified equal width equation and its variants. Commun. Nonlinear Sci. Numer. Simul. **11**, 148–160 (2006)

Yıldırım, Y., Mirzazadeh, M.: Optical pulses with Kundu-Mukherjee-Naskar model in fiber communication systems. Chin. J. Phys. **64**, 183–193 (2020)

Yıldırım, Y., et al.: Highly dispersive optical solitons in birefringent fibers with four forms of nonlinear refractive index by three prolific integration schemes. Optik **220**, 165039 (2020)

Zayed, E.M.E., Al-Nowehy, A.G.: Exact solutions and optical soliton solutions for the (2+ 1)-dimensional hyperbolic nonlinear Schrödinger equation. Optik **127**, 4970–4983 (2016)

Zayed E.M.E., Arnous A.H.: DNA dynamics studied using the homogeneous balance method, Chin. Phys. Lett., 29 (2012).

Zhou, Y., et al.: Periodic wave solutions to a coupled KdV equations with variable coefficients. Phys. Lett. A **308**, 21–36 (2003)